

101 學年四技二專第四次聯合模擬考試 共同考科 數學(B)卷 詳解

數學(B)卷

101-4-B

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
C	A	B	C	D	B	C	A	B	B	D	C	A	A	B	D	A	C	D	B	D	A	D	C	A

1. $\frac{-2}{3} \leq x \leq \frac{3}{4} \Rightarrow (3x+2)(4x-3) \leq 0$
 $\Rightarrow 12x^2 - x - 6 \leq 0 \Rightarrow -12x^2 + x + 6 \geq 0$
 $\Rightarrow -24x^2 + 2x + 12 \geq 0$
 $\therefore (a, b) = (-24, 2) \Rightarrow a + 3b = -24 + 6 = -18$
2. $f(x) = (x-3)^3 + 6(x-3)^2 - 7(x-3) + 9$
 除以 $(x-5)$ 的餘式為 $f(5)$
 $\therefore f(5) = 2^3 + 6 \times 2^2 - 7 \times 2 + 9 = 27$
3. $\begin{cases} 4x + 3y - 9 = 0 \\ x + 5y + 2 = 0 \end{cases} \Rightarrow (x, y) = (3, -1)$
 又夾角為 $\frac{2\pi}{3}$ ，斜率為 $m = \tan \frac{2\pi}{3} = -\sqrt{3}$
 $\therefore L: \sqrt{3}x + y - 3\sqrt{3} + 1 = 0$
4. 由題意可得知通過 $(-12, 0)$ 、 $(8, a)$ 、 $(-2a, -4)$
 $\therefore \frac{a-0}{8-(-12)} = \frac{0-(-4)}{-12-(-2a)} \Rightarrow \frac{a}{20} = \frac{4}{2a-12}$
 $\Rightarrow 2a^2 - 12a - 80 = 0 \Rightarrow a = 10$ 、 -4 (不合)
5. $\frac{a}{2} + \frac{b}{3} + \frac{a}{4} + \frac{b}{9} + \frac{a}{8} + \frac{b}{27} + \dots + \frac{a}{2^n} + \frac{b}{3^n} + \dots$
 $= (\frac{a}{2} + \frac{a}{4} + \frac{a}{8} + \dots) + (\frac{b}{3} + \frac{b}{9} + \frac{b}{27} + \dots)$
 $= \frac{\frac{a}{2}}{1-\frac{1}{2}} + \frac{\frac{b}{3}}{1-\frac{1}{3}} = a + \frac{b}{2} = 5$ ， $\therefore 2a + b = 10$
6. (A) $\cos 27^\circ > \cos 30^\circ = \frac{\sqrt{3}}{2}$
 (B) $\sin 43^\circ = \cos 47^\circ < \cos 33^\circ$
 (C) $\cos 3^\circ = \sin 87^\circ > \sin 85^\circ$
 (D) $\tan 48^\circ = \cot 42^\circ > \cot 48^\circ$
7. 設 $\overline{AC} = x$ ，由 $\triangle ABD$ 及 $\triangle ABC$ 中
 $\Rightarrow \cos B = \frac{5^2 + 7^2 - 3^2}{2 \times 5 \times 7} = \frac{7^2 + 7^2 - x^2}{2 \times 7 \times 7} \Rightarrow x = \sqrt{7}$
8. $\overrightarrow{AB} \cdot \overrightarrow{BC} = -\overrightarrow{BA} \cdot \overrightarrow{BC} = -|\overrightarrow{BA}| |\overrightarrow{BC}| \cos B$
 $= -4 \cdot 6 \cdot \frac{4^2 + 6^2 - 7^2}{2 \cdot 4 \cdot 6} = \frac{-3}{2}$
9. $\cos 200^\circ \cos 100^\circ - \sin(-80^\circ) \sin 20^\circ$
 $= (-\cos 20^\circ)(-\cos 80^\circ) - (-\sin 80^\circ) \sin 20^\circ$
 $= \cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ$
 $= \cos(80^\circ - 20^\circ) = \cos 60^\circ = \frac{1}{2}$

10. 焦點為 $(-1, 4)$ ，短軸在直線 $x = 3$ 上 \Rightarrow 中心為 $(3, 4)$
 又 $2a = 10$ ， $c = 4 \Rightarrow b = 3$

橢圓為 $\frac{(x-3)^2}{25} + \frac{(y-4)^2}{9} = 1$

11. $f(x) = x^3 + ax^2 + bx - 2$ 在 $x = -2$ 時有極大值 -6
 $\therefore f'(x) = 3x^2 + 2ax + b$

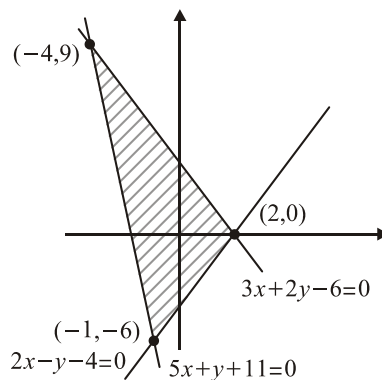
$$\Rightarrow \begin{cases} f'(2) = 12 - 4a + b = 0 \\ f(2) = -8 + 4a - 2b - 2 = -6 \end{cases} \Rightarrow (a, b) = (5, 8)$$

$$a - b = -3$$

12. $\begin{cases} 2x - y - 4 \leq 0 \\ 3x + 2y - 6 \leq 0 \\ 5x + y + 11 \geq 0 \end{cases}$ 所圍出圖形如下

頂點分別為 $(-1, -6)$ 、 $(-4, 9)$ 、 $(2, 0)$

則其面積為 $\frac{1}{2} \begin{vmatrix} -1 & -4 & 2 & -1 \\ -6 & 9 & 0 & -6 \end{vmatrix} = \frac{63}{2}$



13. 偶數排法 = 全部排法 - 奇數排法
 $= 6 \times 6 \times 5 - 5 \times 5 \times 3 = 105$

14. $\sqrt{7-4\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} + (1+\sqrt{2})(\sqrt{3}-\sqrt{6})$
 $= \sqrt{7-2\sqrt{12}} + \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{3}+\sqrt{6})(\sqrt{6}-\sqrt{3})}$
 $+ (\sqrt{3}+\sqrt{6}-\sqrt{6}-\sqrt{12})$
 $= (\sqrt{4}-\sqrt{3}) + (\sqrt{12}-\sqrt{6}) + (\sqrt{3}-\sqrt{12})$
 $= 2 - \sqrt{6}$

15. $\begin{vmatrix} 21 & 24 & 23 \\ 31 & 35 & 33 \\ 41 & 46 & 42 \end{vmatrix} = \begin{vmatrix} 21 & 3 & 2 \\ 31 & 4 & 2 \\ 41 & 5 & 1 \end{vmatrix} = \begin{vmatrix} 21 & 3 & 2 \\ 10 & 1 & 0 \\ 10 & 1 & -1 \end{vmatrix}$
 $= -21 + 20 - 20 + 30 = 9$

$$16. \sin \theta = \frac{-3}{5}, \pi < \theta < \frac{3\pi}{2}$$

$$\Rightarrow \sec \theta + \tan \theta = \frac{-5}{4} + \frac{3}{4} = -\frac{1}{2}$$

$$17. a = 4^{-3} = (2^2)^{-3} = 2^{-6}$$

$$b = (0.25)^{-\frac{3}{2}} = \left(\frac{1}{4}\right)^{-\frac{3}{2}} = (2^{-2})^{-\frac{3}{2}} = 2^3$$

$$c = \left(\frac{1}{8}\right)^{-2} = (2^{-3})^{-2} = 2^6$$

$$d = (0.5)^{0.7} = \left(\frac{1}{2}\right)^{0.7} = 2^{-0.7}$$

$$\Rightarrow c > b > d > a$$

$$18. a \Rightarrow \log_3(-5)^2 \neq 2\log_3(-5) \Rightarrow \text{真數不能為負數}$$

$$b \Rightarrow \log_{\frac{1}{2}} \frac{1}{3} > 1 = \log_{\frac{1}{2}} \frac{1}{2}$$

$$c \Rightarrow \log_{\sqrt{3}} \sqrt{5} = \log_3 5$$

$$d \Rightarrow \log_{\sqrt{3}} 5 > 2 = \log_{\sqrt{3}} (\sqrt{3})^2 = \log_{\sqrt{3}} 3$$

$$19. \text{圓 } x^2 + y^2 - 8x + 6y + 21 = 0$$

$$\Rightarrow C : (4, -3), r = \frac{1}{2} \sqrt{64 + 36 - 84} = 2$$

$$\text{又 } \overline{PA} = \overline{PB} = \sqrt{36 + 4 - 48 + 12 + 21} = 5$$

$$\therefore \triangle ACBP \text{ 面積} = 2\triangle PAC = 2 \times \frac{2 \times 5}{2} = 10$$

$$20. E(x) = 18 \times 4 + 3\left(\frac{1}{3} \times 4 + \frac{2}{3}(-1)\right)$$

$$+ 4\left(\frac{1}{5} \times 4 + \frac{4}{5}(-1)\right) = 74$$

$$21. (\bar{x} - 2S, \bar{x} + 2S) \text{ 占 } 95\%$$

$$\Rightarrow 80 \text{ 分以上佔了 } 2.5\%$$

$$\therefore 600 \times 2.5\% = 15$$

$$22. \text{百分等級為 } \frac{(35000 - 4700)}{35000} \times 100 = 86.6$$

所以小丁的百分等級為 86

$$23. n(S) = 6! = 720, A \text{ 必排首}, C、D \text{ 不相鄰}$$

$\Rightarrow A$ 先置首位, 再排 $B、E、F$, 再將 $C、D$ 置入其中

$$\Rightarrow 3! \times P_2^4 = 6 \times 4 \times 3, \therefore P = \frac{6 \times 4 \times 3}{720} = \frac{1}{10}$$

$$24. \lim_{x \rightarrow 2} \frac{1}{x-2} \left(\frac{1}{x} - \frac{1}{2} \right) = \lim_{x \rightarrow 2} \frac{1}{x-2} \left(\frac{2-x}{2x} \right) = \lim_{x \rightarrow 2} \frac{-1}{2x} = \frac{-1}{4}$$

$$25. \int_{-1}^2 (6x^2 - 2x + 3) dx = 2x^3 - x^2 + 3x \Big|_{-1}^2$$

$$= 2(8+1) - (4-1) + 3(2+1) = 24$$